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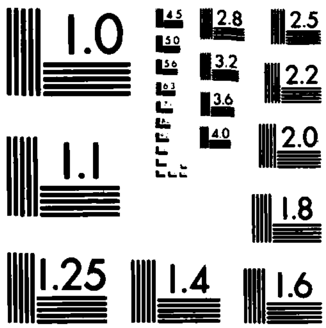

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THEORETICAL RESEARCH
ON
CHARGED PARTICLE BEAM
AND OTHER PLASMA SYSTEMS

FINAL REPORT
Contract N00014-81-C-2355

for
Plasma Physics Division
Naval Research Laboratory
Washington, D.C. 20375

by
Berkeley Scholars, Inc.
P.O. Box 983
Berkeley, California 94701

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INTRODUCTION

This is the final report for Contract N00014-81-C-2355. It covers approximately 40 man-days of research performed by Berkeley Scholars, Inc. from April, 1981 through April, 1982. The research involved the theoretical analysis of charged particle beam and other plasma systems.

The major emphasis of the research was on modeling the free-electron-laser (FEL) oscillator. The approach was to utilize a first order description for the evolution of relativistic electron beam pulses in a cavity with an applied wiggler magnetic field to develop a model for the modal electromagnetic energy in a FEL oscillator. In the following TECHNICAL DISCUSSION we present the details of our research.

TECHNICAL DISCUSSION

During the performance period, Berkeley Scholars, Inc. was funded for approximately 40 man-days to examine several areas of charged-particle beam research and other plasma systems. These included relativistic charged-particle beam propagation and the free-electron-laser. The free-electron-laser is of great interest due to its potentially high efficiency (~20%), high power operation (few MW), and continuous frequency tunability. The major emphasis of our research was on modeling the free-electron-laser (FEL) oscillator. Discussion and details of our research follows:

The FEL Oscillator

Figure 1 is a pictorial description of a free-electron-laser at $t = 0$. N_o is the average density of the incoming electron beam pulses of width L_b and separation L_c. The cavity mirrors are at $z = 0$ and $z = L$ with the wiggler magnetic field applied between $z = L_o$ and $z = L_o + L_w$. For a relativistic electron the equation of motion is

$$\frac{d}{dt}(m\gamma\underline{v}) = -e\left(\underline{E} + \frac{\underline{v}\times\underline{B}}{c}\right)$$

where

$$\gamma = \left(1 - \frac{v^2}{c^2}\right)^{-\frac{1}{2}} .$$

Introducing the vector potential $\underline{A}$ and the scalar potential Φ such that

$$\underline{B} = \nabla \times \underline{A} \text{ and } \underline{E} = -\nabla\Phi - \frac{1}{c}\frac{\partial \underline{A}}{\partial t}$$

and assuming for the canonical model of the FEL

$$\underline{A} = A_x(z,t)\underline{e}_x + A_y(z,t)\underline{e}_y$$

$$\Phi = \Phi(z,t)$$

yields

$$m\gamma\underline{v}_\perp = \frac{e\underline{A}}{c} \quad \text{where } \underline{V}_\perp = v_x\underline{e}_x + v_y\underline{e}_y$$

and in the tenuous beam limit where Φ is negligible

$$\dot{v}_z = -\left(\frac{e}{mc\gamma}\right)^2\left(\frac{v_z}{c^2}\frac{\partial}{\partial t} + \frac{\partial}{\partial z}\right)\frac{\underline{A}^2}{2} .$$

Linearizing to first order with

$$\underline{A} = \underline{A}_o + \underline{A}_1$$

$$z = z_o + v_o t + z_1$$

$$\underline{A}_o = -\frac{B_w}{k_w}\left(\underline{e}_x \cos k_w z + \underline{e}_y \sin k_w z\right) H(z-L_o)H(L_o+L_w-z)$$

where B_w and k_w are the amplitude and wavevector of the wiggler field and H is a heaviside function yields

$$\ddot{z}_1 = -\left(\frac{e}{mc\gamma_o}\right)^2\left(\frac{v_o}{c^2}\frac{\partial}{\partial t} + \frac{\partial}{\partial t}\right)\underline{A}_o \cdot \underline{A}_1 \quad . \qquad (1)$$

Expanding $\underline{A}_1$ in terms of the vacuum modes of the cavity

$$\underline{A}_1 = \sum_{n=-\infty}^{\infty} \underline{a}_n(t) e^{-i\omega_n t} \sin k_n z \qquad (2)$$

where $\underline{a}_{-n} = \underline{a}_n^*$, $\omega_n = \frac{n\pi c}{L}$ and $k_n = \frac{|n|\pi}{L}$

and substituting equation (2) into equation (1) yields for $L_o < z < L_o + L_w$

$$\ddot{z}_1 = \frac{1}{4}\frac{B_w}{k_w}\left(\frac{e}{mc\gamma_o}\right)^2 \left[\sum_{n=1}^{\infty}\left(k_n + k_w - \frac{v_o}{c^2}\,\omega_n\right)\left(\underline{e}_x - i\underline{e}_y\right)\cdot \underline{a}_n e^{i(k_n+k_w)z_o + i\mu_n t}\right.$$

$$\left. - \sum_{n=-\infty}^{-1}\left(k_n + k_w + \frac{v_o}{c^2}\,\omega_n\right)\left(\underline{e}_x + i\underline{e}_y\right)\cdot \underline{a}_n e^{-i(k_n+k_w)z_o - i\mu_n t}\right]$$

where $\mu_n = (k_n + k_w)v_o - |\omega_n|$ and $|\dot{\underline{a}}_n/\mu_n| \ll 1$. Since z_1 and $\dot{z}_1$ vanish if the electron in question has not entered the wiggler

$$z_1(t) = \int_o^t d\tau\,(t-\tau)\,\ddot{z}_1(\tau)\,H(z_o + v_o\tau - L_o)\,H(L_o + L_w - z_o - v_o\tau)$$

and it follows that

for $t < \frac{L_o - z_o}{v_o}$: $z_1(t) = 0$,

for $\frac{L_o - z_o}{v_o} < t < \frac{L_o + L_w - z_o}{v_o}$:

$$z_1(t) = -\frac{1}{4}\frac{B_w}{k_w}\left(\frac{e}{mc\gamma_o}\right)^2\left[\sum_{n=1}^{\infty}\left(k_n+k_w-\frac{v_o}{c}\omega_n\right)\left(\underline{e}_x-i\underline{e}_y\right)\cdot\underline{a}_n e^{i(k_n+k_w)z_o}\right.$$

$$\left.\frac{1}{\mu_n^2}\left\{e^{i\mu_n t}-\left[i\mu_n\left(t-\frac{L_o-z_o}{v_o}\right)+1\right]e^{i\mu_n\frac{L_o-z_o}{v_o}}\right\}\right]$$

$$-\frac{1}{4}\frac{B_w}{k_w}\left(\frac{e}{mc\gamma_o}\right)^2\left[\sum_{n=-\infty}^{-1}\left(k_n+k_w+\frac{v_o}{c}\omega_n\right)\left(\underline{e}_x+i\underline{e}_y\right)\cdot\underline{a}_n e^{-i(k_n+k_w)z_o}\right.$$

$$\left.\frac{1}{\mu_n^2}\left\{e^{-i\mu_n t}-\left[-i\mu_n\left(t-\frac{L_o-z_o}{v_o}\right)+1\right]e^{-i\mu_n\frac{L_o-z_o}{v_o}}\right\}\right], \quad (3)$$

$$\text{and for } t > \frac{L_o+L_w-z_o}{v_o} : \quad z_1(t) = z_1\left(\frac{L_o+L_w-z_o}{v_o}\right) .$$

The above calculations are for an individual electron within the beam pulse. Therefore the beam density is defined by

$$N = \frac{1}{\sigma}\sum_j \delta(z-z_j(t))$$

where σ denotes the cross-sectional area of the electron beam. The average density in the absence of the high frequency field is then

$$N_o = \frac{1}{\sigma} < \sum_j \delta(z - z_{oj} - v_o t) >$$

where <...> denotes the ensemble average. Consequently, the equation for the first order vector potential is

$$\frac{\partial^2 \underline{A}_\perp}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 \underline{A}_\perp}{\partial t^2} = -\frac{4\pi}{c}\underline{J}_\perp \approx \frac{4\pi e}{c}\left[N-N_o\right] \underline{v}_{\perp o} \ . \qquad (4)$$

Substituting equation (2) into equation (4)

$$\sum_{m=-\infty}^{\infty} \frac{1}{c^2}\left(2i\omega_m \dot{\underline{a}}_m - \ddot{\underline{a}}_m\right) e^{-i\omega_m t} \sin k_m z = \frac{4\pi e}{c}\left[N - N_o\right] \underline{v}_{\perp o}$$

where $|\omega_m \dot{\underline{a}}_m| >> |\ddot{\underline{a}}_m|$ and solving

$$L \frac{i\omega_n}{c^2} \dot{\underline{a}}_n e^{-i\omega_n t} = \frac{4\pi e}{c}\int_o^L dz \sin k_n z \left[N - N_o\right] \underline{v}_{\perp o} \ . \qquad (5)$$

Linearizing and inserting equation (3) into equation (5)

$$\dot{\underline{a}}_m - \underline{b}_m = \frac{2\pi i e^2 B_w(\underline{e}_x + i\underline{e}_y)}{mc\gamma_o k_w L} \frac{1}{4} \frac{B_w}{k_w} \left(\frac{e}{mc\gamma_o}\right)^2$$

$$\int_{L_o - v_o t}^{L_o + L_w - v_o t} dz_o N_o(z_o, 0) \sum_{n=1}^{\infty}\left(k_n + k_w - \frac{v_o}{c}\omega_n\right)\left(\underline{e}_x - i\underline{e}_y\right)\cdot \underline{a}_n$$

$$e^{i(k_n-k_m)z_o + i(\mu_n - \mu_m)t} \frac{1}{\mu_n^2}\left\{1 - \left[i\mu_n\left(t - \frac{L_o + z_o}{v_o}\right) + 1\right] e^{-i\mu_n\left(t - \frac{L_o - z_o}{v_o}\right)}\right\} \qquad (6)$$

with

$$\underline{b}_n = \frac{2\pi e^2 B_w(\underline{e}_x + i\underline{e}_y)}{m\gamma_o \omega_n k_w L\sigma} \int_{L_o}^{L_o + L_w} dz \left[\sum_j \delta(z - z_{oj} - v_o t) - \sigma N_o\right]$$

$$e^{-i(k_n + k_w)z + i\omega_n t} \qquad (7)$$

and $\underline{b}_{-n} = \underline{b}_n^*$ for $n \geq 1$.

Equations (6) and (7) suggest writing

$$\underline{a}_m = (\underline{e}_x + i\underline{e}_y)\alpha_m e^{-i\mu_m t}$$

$$\underline{b}_n = (\underline{e}_x + i\underline{e}_y)\beta_n e^{-i\mu_n t} \quad . \tag{8}$$

Forming the scalar product of equation (6) with $(\underline{e}_x - i\underline{e}_y)$ yields

$$\dot{\alpha}_m = \beta_n + M_{mn}\alpha_n$$

or in matrix notation

$$\dot{\alpha} = \beta + M\alpha \tag{9}$$

where

$$M_{mn} + i\mu_m\delta_{mn} = \pi i\left(\frac{eB_w}{mc\gamma_o k_w}\right)^2 \frac{e^2}{mc^2\gamma_o} \int_{L_o - v_o t}^{L_o + L_w - v_o t} dz\ N_o(z,0)$$

$$\left(k_n c + k_w c - \frac{v_o\omega_n}{c}\right)\frac{1}{\mu_n^2} e^{i(k_n - k_m)z}\left\{1 - \left[i\mu_n\left(t - \frac{L_o - z}{v_o}\right) + 1\right]e^{-i\mu_n\left(t - \frac{L_o - z}{v_o}\right)}\right\}. \tag{10}$$

The objects of physical interest are the $\langle|\alpha_j|^2\rangle$ which measure the energy per line.

$$\frac{1}{8\pi}\int_o^L dz(\underline{E}^2 + B^2) = \frac{1}{\pi}\int_o^L dz\left[\left(\frac{1}{c}\frac{\partial \underline{A}}{\partial t}\right)^2 + \left(\frac{\partial \underline{A}}{\partial z}\right)^2\right] = \frac{L}{\pi}\sum_{n>o}|\alpha_n|^2$$

Defining the matrix

$$C = \langle \alpha\tilde{\alpha} \rangle$$

where ~ denotes the Hermitian conjugate we can then formally solve equation (9) for C.

$$\alpha(t) = \int_0^t d\tau P(t,\tau)\beta(\tau)$$

where $P(t,\tau) = Q^{-1}(t)Q(\tau)$, $\dot{Q} = -QM$, $Q(0) = I$, and I is the identity matrix. Note that

$$\dot{P}(t,\tau) = M(t)P(t,\tau) \text{ and } P(t,t) = I \ .$$

Therefore

$$C(t) = \int_0^t d\tau \int_0^t d\tau' P(t,\tau) \langle \beta(\tau)\tilde{\beta}(\tau') \ \tilde{P}(t,\tau')$$

and

$$\dot{C}(t) = MC + C\tilde{M} + D$$

where

$$D = \int_0^t d\tau \langle \beta(t)\tilde{\beta}(\tau) \rangle \tilde{P}(t,\tau) + \int_0^t d\tau P(t,\tau) \langle \beta(\tau)\tilde{\beta}(t) \rangle \ . \qquad (11)$$

Clearly

$$\text{TRACE } C = \frac{\pi}{L} \text{ (ensemble average electromagnetic energy)}$$

and

$$(\text{TRA}\dot{\text{C}}\text{E } C) = \text{TRACE } (M+\tilde{M})C + \text{TRACE } D \ . \qquad (12)$$

Consequently the Hermitian par. of M determines the total gain.

From equations (7) and (8)

$$
\begin{aligned}
\sigma\omega_n\omega_m\left(\frac{m\gamma_o k_w L}{2\pi e^2 B_w}\right)^2 \langle\beta_n(t)\,\beta_m^*(\tau)\rangle
&\underset{t>\tau}{=} \int_{L_o-v_o\tau}^{L_o+L_w-v_o t} dz_o N_o(z_o,0)\, e^{i(k_m-k_n)z_o}\, H\left[L_w-v_o(t-\tau)\right] \\
&\underset{t<\tau}{=} \int_{L_o-v_o t}^{L_o+L_w-v_o\tau} dz_o N_o(z_o,0)\, e^{i(k_m-k_n)z_o}\, H\left[L_w-v_o(\tau-t)\right] .
\end{aligned}
\tag{13}
$$

For $t>\tau$ equation (13) involves at most one pulse and requires that the pulse be in the wiggler in order to emit. From equations (11) and (13) when $(k_m-k_n)L_b<1$

$$
D_{mn} \approx \sum_{j\geq 1} 2\frac{L_w}{c}\,\frac{1}{\sigma\omega_n\omega_m}\left(\frac{2\pi e^2 B_w}{m\gamma_o k_w L}\right)^2 \bar{N}L_b\, e^{-i(k_m-k_n)(jL_c+\frac{1}{2}L_b)}\, H(v_o t-L_o-jL_c)\, H(L_o+L_w+jL_c-v_o t) \tag{14}
$$

where

$$
\bar{N}L_b = \int_{-jL_c-L_b}^{-jL_c} dz\, N_o(z,0)
$$

and D is a positive definite matrix. Therefore from equations (10), (12), and (14) the evolution of the ensemble average of the electromagnetic energy may be calculated.

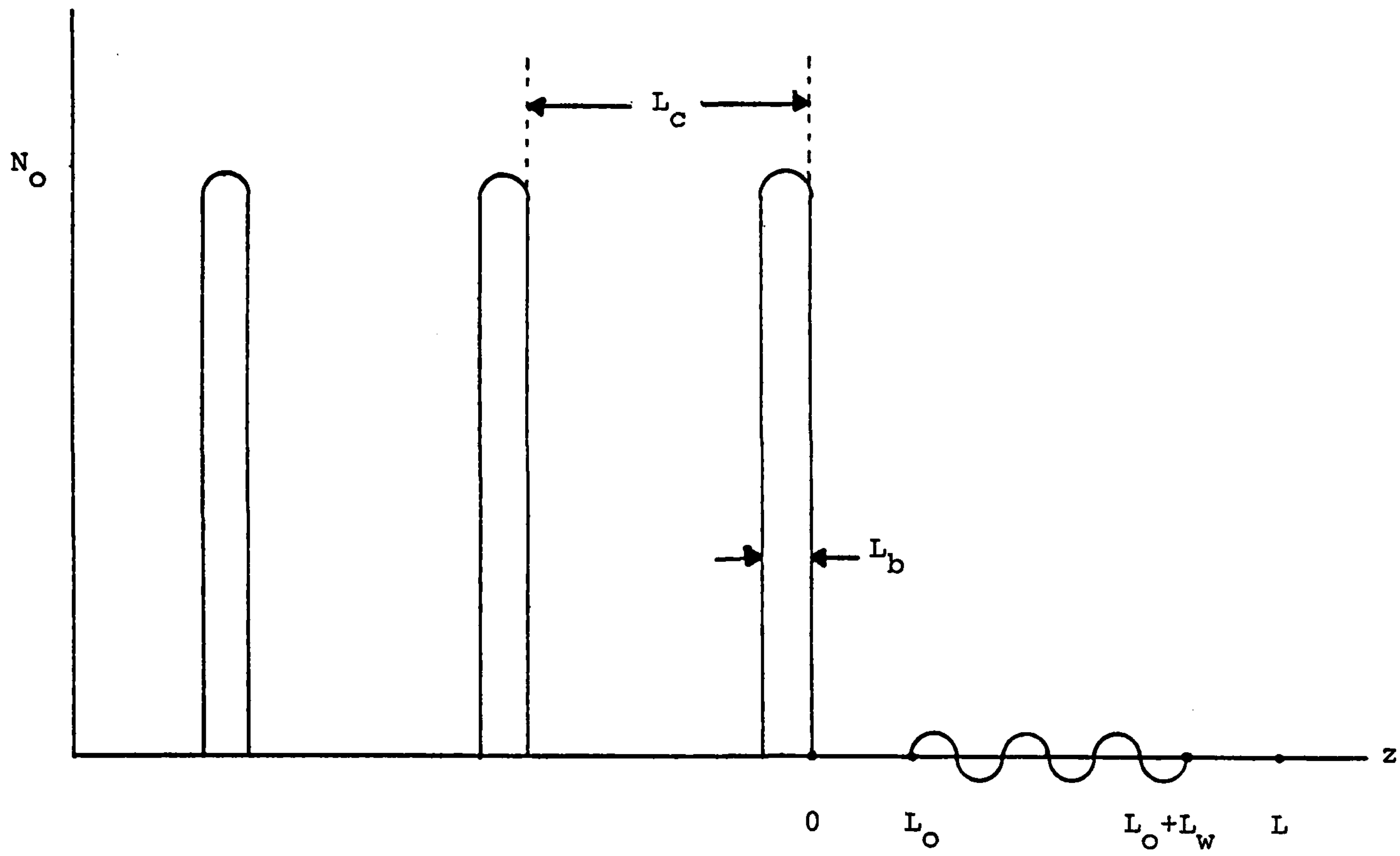

Figure 1. Pictorial description of a free-electron-laser at t = 0.

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